EMPIRICAL ANALYSIS OF VALUE AT RISK AND EXPECTED SHORTFALL IN PORTFOLIO SELECTION PROBLEM

A Thesis

by

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ABSTRACT

Safety first criterion and mean-shortfall criterion both explore cases of assets allocation with downside risk. In this paper, I compare safety first portfolio selection problem and mean-shortfall portfolio optimization problem, considering risk averse investors in practice. Safety first portfolio selection uses Value at Risk (VaR) as a risk measure, and mean-shortfall portfolio optimization uses expected shortfall as a risk measure, respectively. VaR is estimated by implementing extreme theory using a semi-parametric method. Expected shortfall is estimated by two nonparametric methods: a natural estimation and a kernel-weighted estimation.

I use daily data on three international stock indices, ranging from January 1986 to February 2012, to provide empirical evidence in asset allocations and illustrate the performances of safety first and mean-shortfall with their risk measures. Also, the historical data has been divided in two ways. One is truncated at year 1998 and explored the performance during tech boom and financial crisis. the mean-shortfall portfolio optimization with the kernel-weighted method performed better than the safety first criterion, while the safety first criterion was better than the mean-shortfall portfolio optimization with the natural estimation method.

DEDICATION

To my mother

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1. INTRODUCTION

A theory of investment, namely modern portfolio theory (MPT), was introduced by Markowitz in 1952. Markowitz developed this MPT to explain how investors could obtain the optimal portfolio. By setting the proportions of different assets, an investor achieves either the maximum expected return for a low level of risk, or the minimum risk associated with a desired level of expected return. MPT takes standard deviation of returns as the measure of risk and for a given expected return seeks to minimize the total variance of linearly combined asset returns. A weakness of MPT is that it requires strict assumptions such as rational investors and efficient market. However, growing evidence suggests that the theoretical assumptions required for MPT are not always met in reality (see Shleifer 2000; Koponen 2003). For instance, investors may be concerned with the limited downside risk. If so, investors are likely to choose a portfolio that differs from the portfolio that is chosen by MPT. Downside risk focuses on the standard deviation of returns' dispersion below a given target level, and it is also named downside deviation. Downside risk is an appealing measure since it is also consistent with intuitive notions that investors desire relatively low risk with high return. There are several methods related to downside risk. One approach is Roy's (1952) safety first criterion. This method is designed to fulfill the investors' demand for a limitation on downside risk. The safety first criterion was originally operationalized by the Chebyshev bound. However, the original formulation of safety first by Roy led to sharp discontinuities in portfolio choice. Arzac and Bawa (1977) brought several improvements by bringing in consideration of borrowing and lending. Another approach that allows risk averse investors to choose optimal portfolio is the mean-shortfall optimization criterion developed upon mean-variance optimization criterion. The mean-shortfall optimization criterion has an explicit explanation regarding the expected shortfall's properties which has been presented in Bertsimas, Lauprete and Samarov's (2004) paper.

The mean-shortfall optimization problem is based on a risk measure called shortfall which can be decomposed into expected return and expected shortfall. Some properties of the shortfall have been shown in Uryasev and Rockafelar (1999), Tasche (1999), and Bertsimas, Lauprete and Samarov (2004.) The advantage of the mean-shortfall over the mean-VaR optimization problem is that, because of the convexity of the portfolio selection problem under mean-shortfall, it is efficiently solvable as a linear portfolio selection problem (see Bertsimas, Lauprete and Samarov 2004).

In this paper, I examine the use of alternative risk measures in the safety first portfolio selection problem and mean-shortfall selection problem. These risk measures are Value at Risk (VaR), in safety first, and expected shortfall, in mean-shortfall. I am interested in how these alternative measures of risk will impact portfolio allocations. In addition, I examine how portfolios perform in a tech boom or a financial crisis when selected by safety first and mean-shortfall respectively under these two risk measures.

Historical data are used, including daily returns on several market indices (the Hang-Seng Index, Nikkei, and S&P 500) for the last decade. The model used to address VaR under safety first is a semi-parametric mode from extreme value theory. Expected shortfall under mean-shortfall is computed by two estimations, a natural estimation as illustrated by Bertsimas, Lauprete and Samarov's (2004) and a kernel-weighted polynomial estimation.

The rest of this paper is organized as follows. Section 2 reviews the definitions of the risk measures. Section 3 outlines the process of portfolio selection problems. Section 4 describes the methods to compute VaR and expected shortfall. Section 5 presents the data set and statistics of the variables. Portfolios considering limited downside risk are constructed by different methods and then compared. Section 6 concludes the paper.

2. DEFINITIONS

2.1 Coherent Risk Measure

A coherent risk measure is defined by Artzner et al (1999) as follows: consider a set V of real-valued random variables; a function $\rho: V \to R$ is called a coherent risk measure if it is

(i) Monotonous: $X \in V, X \geq 0 \Rightarrow \rho(X) \leq 0$,

(ii) Sub-additive: $X, Y, X + Y \in V \Rightarrow \rho(X + Y) \leq \rho(X) + \rho(Y)$,

(iii) Positively homogeneous: $X \in V, h > 0, hX \in V \Rightarrow \rho(hX) = h\rho(X)$,

(iv) Translation invariant: $X \in V, a \in R \Rightarrow \rho(X + a) = \rho(X) - a$.

2.2 Value at Risk

Value at Risk did not draw much attention until the stock market suffered from the first major financial crisis in late 1980s. After the market crash in 1987, it becomes a widely used measure of risk. It was adopted by regulators in the Basel Accords as a required measure for banks to report, and it is, therefore, included in the notes of financial statement by major banks and dealers.

VaR is a threshold value that indicates the possible maximum loss on a specific portfolio over a given time horizon at a certain confidence level.

Given a random variable X as return and some $(1-\alpha)\epsilon(0,1)$ as confidence level, the VaR at the confidence level $(1-\alpha)$ is given by the negative value of the level α-quantile when assuming continuous distribution for X, i.e.

$$VaR_{\alpha}(X) = -q_{\alpha}(X) = -\inf\{x \in R : P[X \leq x] > \alpha\}. \tag{1}$$

Intuitively, VaR gives the smallest x such that the probability of loss exceeds x is smaller than or equivalent to α.

For example, suppose that an investor is deciding between two portfolios, portfolio A and portfolio B. He is provided the information that the VaR at the 99% confidence level of portfolio A is 1 billion and that of portfolio B is 1.1 billion. Note that although VaR is representing a loss on portfolio, it is reported as a positive number. A negative VaR indicates that the specific portfolio has a certain probability of making a profit over the given time horizon (usually one day). Thus, given the information regarding VaR, the investor may end up investing in portfolio A since portfolio B is more risky (has a higher loss that occurs with a 1% probability) than portfolio A. However, VaR does not contain any information outside the confidence level. For instance, in the above example portfolio B is considered riskier because it has a higher VaR than portfolio A at the 1% probability level, but portfolio B's maximum possible loss could be 1.1 billion whereas portfolio A could have a maximum possible loss much greater than 1.1 billion. Thus the characterization of risk by VaR is limited in scope and potentially misleading. Other criticisms of VaR are that it is not subadditive in the sense that it discourages diversification and is not a coherent risk measure (See Acerbi and Tasche 2001).

2.3 Expected Shortfall

Expected shortfall is defined as the expected loss given the loss exceeds VaR. Given X as the payoff of the portfolio at some future time and some $\alpha \epsilon (0, 1)$ as probability level, the ES requires a quantile-level q_α such that ES is the expected loss of portfolio when a loss is occurring at or below the q_α-quantile, i.e.

$$ES_\alpha(X) = -\inf\{E[X|A]: P(A) \geq \alpha\}, \tag{2}$$

If X exhibits a continuous distribution, then equation (2) can be expressed as:

$$ES_\alpha(X) = -E[X|X \leq q_\alpha(X)]. \tag{3}$$

The above expression of expected shortfall is also considered as the definition of tail conditional expectation by Artzner et al (1999). It is shown by Artzner et al (1999) that the tail conditional expectation is a coherent risk measure when X exhibits a continuous distribution.

Given an example to understand the expected shortfall, suppose at the beginning of an investment we paid 100 for the portfolio and the probability of event in each case is listed as follows: 10% of -100 profit event, 20% of -50 profit event, 20% of 0 profit event and 40% of 40 profit event. To calculate $ES_{0.20}$, the expectation of payoff in the worse 20 out of 100 cases, 10 cases of -100 profit event and another 10 cases of -50 profit event are considered. The formula for this expected value is:

$$\frac{\frac{10}{100}\times(-100)+\frac{10}{100}\times(-50)}{\frac{10+10}{100}} = -75.$$

3. PORTFOLIOS SELETION METHODS

3.1 Safety First Portfolio Selection

Arzac and Bawa (1977) explore the investors' behavior under limited downside risk with the ability to lend and borrow. Thus, a lexicographic form of Roy's (1952) safety first principle is:

$$\max_{\gamma_j, b}(\pi, \mu) \tag{4}$$

subject to $\sum_j \gamma_j V_j - b = W,$

where

$$\pi = \begin{cases} 1, & if\ P = Pr\left\{\sum_j \gamma_j X_j - br \le s\right\} \le \alpha \\ 1 - P, & otherwise \end{cases}$$

and

$$\mu = \sum_j \gamma_j \overline{X_j} - br.$$

Here, $\gamma = (\gamma_1, \dots, \gamma_n)$ denotes the proportion of assets allocation in portfolio. V_j , X_j are the initial and final values respectively of asset j. W is the wealth an investor held. An investor could borrow b at an interest rate $r - 1$ where if b is negative then it denotes lending. The critical value of wealth is s such that the investor would never want his final wealth fall below. Intuitively, if the probability of final wealth falls below s does not exceed α, then investor would only care about the expected return μ. Define a material risk as the asset not fulfilling the condition for $\pi = 1$. By substituting budget constraint and q_α(R) into the condition equation for $\pi = 1$, it reaches a meaningful characteristic of a material risk:

$$q_\alpha(R) < r + \frac{s-Wr}{W+b}. \tag{5}$$

where $R = \frac{X}{V}$. Thus, investors will only invest in favorable assets and (4) is rewritten as:

$$\max_{\gamma_j} \frac{\bar{R}-r}{r-q_\alpha(R)}, \tag{6}$$

where $\bar{R}$ is the average rate of return of the risky assets.

3.2 Mean-shortfall Portfolio Selection

Mean-shortfall is an alternative quantile-based downside-risk measure for the portfolio selection problem (see Bertsimas, Lauprete and Samarov 2004). The widely known mean-covariance portfolio selection is based on utility maximization of investors. Particularly, investors have two types of utility functions: 1) increasing utility functions and 2) increasing and concave utility functions on asset returns. However, mean-covariance portfolio selection is restricted by the requirements on asset returns' distributions in the sense that either normal distributions or elliptically symmetric distributions have to be imposed. Focusing on the risk averse investors' portfolio choice, the mean-shortfall optimization problem relaxes the assumption on returns' distributions by using nonparametric estimation methods of expected shortfall. Also, the mean-shortfall minimization problem is drawn under increasing and concave utility functions. The result of such a portfolio is proved to be non-dominated among the portfolios with fixed proportions by Levy and Kroll (1978) and Levy (1992).

The principle of mean-shortfall is defined as:

$$\min S_{\alpha}(\gamma) \tag{7}$$

subject to

$$\gamma'\mu = r_p,\ e'\gamma = 1$$

where $\gamma = (\gamma_1, \dots, \gamma_n)'$ is the assets allocation of the portfolio, e is a n-dimension column vector of 1s, and $\mu = E(R)$ denotes a n-dimension column vector of expected returns of the n assets contained in the portfolio. Here, $S_{\alpha}(\gamma)$ is the modified shortfall such that it takes the expected returns separately into consideration. Minimizing the modified shortfall is equivalent to minimizing the ES under a given assets return. The modified shortfall at risk level α is defined as follow:

$$S_{\alpha}(\gamma) = \mu'\gamma - E[R'\gamma | R'\gamma \leq q_{\alpha}(R'\gamma)]. \tag{8}$$

Recall the definition of ES_{α}, the modified expected shortfall can be expressed in another form as:

$$S_{\alpha}(W) = \mu'\gamma + ES_{\alpha}. \tag{9}$$

Note that $W = R'\gamma$. From this expression, it can be seen that the modified expected shortfall does not satisfy the coherent risk measure properties (i) and (iv).

4. ESTIMATION METHODS

4.1 Implementing Extreme Value Theory in Value at Risk

Applying extreme value theory (EVT) in VaR methodologies has become increasingly popular as it is a way to avoid the counterfactual assumption that returns are normally distributed. It is well known that stock returns are fat tailed and hence assumptions of normality, while convenient, are problematic at best. In the field of statistics, EVT focuses on the maximum and minimum values of a random process over a given time period. Several applications to various areas are mentioned in Galambos, Lechner and Simiu (1994)'s book which utilize the EVT. Reiss and Thomas (1997) wrote a book about EVT from applied statistics' point of view. EVT was used to describe exchange rates by Koedijk, Schafgans and de Vries (1990) and to describe stock returns by Jansen and de Vries (1991). Longin (1996) showed that the limiting distribution of extreme returns within a long time zone is independent of the distribution of returns itself. Thus, the approach based on extreme values to compute VaR is appropriate under both usual and financial crises conditions. Moreover, Longin (1996 and 2000) generates similar results for U.S. univariate studies with Jansen and de Vries (1991) and Loretan and Phillips (1994). Longin and Solnik (2001) further explores the multivariate distribution tails using EVT for financial series and used a Monte Carlo simulation method that is inherited from Jansen and de Vries (1991).

Jansen (2001) and Jansen, Koedijk and de Vries (2000) demonstrates safety first portfolio selection using EVT for estimating tail index (see de Haan, Jansen, Koedijk, de Vries 1994). This paper also proposed an approach to explore the VaR and fat tail index. The idea using EVT is outlined below.

The probability of the maximum $M_n = \max(X_1, X_2, \ldots)$ of the first n i.i.d. random variables such that it does not exceed a given value x is carried out that $P(M_n < x) = F^n(x)$. The cumulative distribution function $F^n(x)$ of the order statistic M_n converges to a limiting distribution $G(x)$ concerned by EVT when simply normalized and n is large. Since the stock returns are fat tailed, the $G(x)$ considered here is characterized by a lack of some higher moments and is formed as:

$$G(x) = \begin{cases} 0, & x < 0 \\ \exp(-x)^{-1/\gamma} = \exp(-x)^{-\alpha}, & x > 0. \end{cases} \tag{10}$$

Note that $\gamma > 0$, and α is the tail index. In this paper, as Jansen (2001) previous suggested, I assume that the distribution converges to the limiting distribution $G(x)$ given above.

To compute α, Hill's (1975) moment estimator is used by calculating the order statistics for the random variable. Then I get $\hat{\alpha}$, the Hill (1975) estimator, as:

$$\widehat{\frac{1}{\alpha}} = \frac{1}{m} \sum_{i=1}^{m} \left[\log\left(\frac{X_{(n+1-i)}}{X_{(n-m)}}\right)\right]. \tag{11}$$

Here, m is the number of upper order statistics obtained and n is the total number of observations. It is proved by Mason (1982) that $\hat{\alpha}$ is a consistent estimator of α. The choice of the order statistics m can be arbitrary. To estimate the quantile-q, Jansen (2001) uses the following equation based on ideas of Dekkers et al. (1989), de Haan et al. (1994) and Jansen and de Vries (1991):

$$\widehat{q_p} = X_{(n-m)} \left(\frac{m}{pn}\right)^{\widehat{\frac{1}{\alpha}}}. \tag{12}$$

4.2 Estimation of Modified Expected Shortfall

The modified shortfall is constructed by two parts, expected return and expected shortfall. The expected return is estimated by the sample mean given certain portfolio allocations. The expected shortfall is calculated by two nonparametric approaches. One approach simply imposes a natural estimator of the ES. Another approach uses kernel polynomial estimator to get the ES.

Given a sample of T returns on the n assets $R_1, \dots, R_T$, period t's portfolio return is simply $R_t(\gamma) = R_t'\gamma$ under the certain asset allocation γ. Furthermore, let $\bar{R} = (\overline{R^1}, \dots, \overline{R^n}\ )'$ be an n column vector that denotes the sample mean of T returns on n assets. Thus, the expected return, $\mu'\gamma$, is estimated by $\gamma'\bar{R}$.

4.2.1 The Natural Estimator of ES

One way to estimate the ES is to propose a natural estimator of ES_α. First, we calculate the order statistics for $R_t(\gamma)$ as $R_{(1)}(\gamma) \leq \cdots \leq R_{(T)}(\gamma)$, and define $m = \alpha T$. We obtain a non-parametric estimator of $s_\alpha(\gamma)$ which does not depend on any distribution assumptions on R such that

$$\hat{s}_\alpha(\gamma) = \gamma'\bar{R} - \frac{1}{m}\sum_i^m R_{(i)}(\gamma) \tag{13}$$

Here, $ES_\alpha = -\frac{1}{m}\sum_i^m R_{(i)}(\gamma)$ is the average loss of m worst cases.

4.2.2 Nonparametric Kernel-weighted Estimation

First, X_i is used to denote the return process and Y_i to denote the explanatory variable which includes the lags of X_i. Thus, $F(\cdot\,|y)$ is the distribution function of X_i, given $Y_i = y$. Define $q_\alpha = -F^{-1}(\alpha|y) = -\inf\{x\colon F(x|y) \geq \alpha\}$ and $Q_{ES} = -E(X_i\,|X_i \leq q_\alpha, Y_i = \mathrm{y})$. Estimating VaR and ES is equivalent to estimate the q_α and Q_{ES} respectively. Q_{ES} then is as following:

$$Q_{ES} = \alpha^{-1}\int_{-\infty}^{q_\alpha} x\, f(x|y)dx = \alpha^{-1}E[X_i \cdot 1(X_i \leq q_\alpha)|Y_i = y)], \tag{14}$$

where $1(\cdot)$ is the indicator function. Note that information about q_α is limited, nevertheless an alternative method can be used that Q_{ES} estimated directly through kernel smoothing. If $\widehat{q_\alpha}$ is given and now consider kernel-weighted local l-th polynomial estimator introduced by Fan and Yao (2003) of Q_{ES}. Now define $\hat{Q} = \widehat{\theta_0}$ and

$$\left\{\widehat{\theta_j}, 0 \leq j \leq l\right\} = \tag{15}$$

$$\min_{\{\theta_j:0\leq j\leq l\}} \sum_{i=1}^n [\alpha^{-1}X_i G(\tfrac{\widehat{q_\alpha}-X_i}{h_0}) - \sum_{j=0}^l \theta_j (Y_i - y)^j]^2 K_h(Y_i - y).$$

Thus,

$$\hat{Q} = \frac{1}{nh}\sum_{i=1}^{n} W_l\left(\frac{Y_i - y}{h}\right) X_i G\left(\frac{\widehat{q_\alpha} - X_i}{h_0}\right)\alpha^{-1}, \tag{16}$$

where $W_l(u) = e_1' H^{-1}(1, uh, \dots, (uh)^l)' K(u)$ with $H = diag(1, h, \dots, h^l)$ satisfying $\frac{1}{nh}\sum_{i=1}^{n} W_l(\frac{Y_i - y}{h}) = 1$ and e_1 be the $(l+1)$ $-$dimesion vector $(1,0, \dots, 0)'$.

Now consider the local polynomial estimator $\widehat{q_\alpha}$ solves the following problem:

$$\frac{1}{nh}\sum_{i=1}^{n} W_l\left(\frac{Y_i - y}{h}\right) G\left(\frac{\widehat{q_\alpha} - X_i}{h_0}\right) = \alpha \tag{17}$$

In this paper, kernel-weighted local constant estimator is used. The mean of sample portfolio returns is used as the explanatory variable. Since there exists only one explanatory variable, only one bandwidth of the kernel functions on X_i need to be found. The smoothing parameters (bandwidths) of kernels on X_i and Y_i are both chosen by rule-of-thumb methods since it is the most commonly used method in practice.

5. DATA ANALYSIS

In this section, it is assumed that a risk averse investor is willing to invest in a portfolio consisting of three stock indices: The Hang-Seng Index (a market-weighted index) from Hong Kong, The Nikkei Stock Index from Japan, and the S&P 500 Index from the United States. To explore how risk measures perform in fixed and diversified portfolio allocations, corner solutions are ignored. This paper only considers the portfolios with weight of at least 10% for each of the three stock indices. Moreover, portfolio weights are ten percent of a portfolio. Following this method, a total of thirty-six portfolios are listed as all the possible portfolios being examined at one time.

The daily data of price indexes of the three indices as well as the daily data of Japan Yen, Hong Kong $ and US $ are downloaded from DataStream. By definition, the stock return can be calculated from the logarithm of the price index, namely $X_i = \ln(\frac{P_i}{P_{i-1}})$. X_i is the stock return in period i. P_i denotes the price index of period i, and P_{i-1} denotes the lag of P_i (the price index of period i-1). Suppose the risk averse investor use US $ to invest, the exchange rates is computed similarly to the logarithm of the price index. All the data displayed below is in US $.

The historical data covers from January 1986 to February 2012. This timeline covers the early 2000s tech boom, specifically from year 1999 to 2003, and the financial crisis of

Table 1

Descriptive statistics for the 1986-1998 daily return data

	Mean (x1000)	Std Dev(x10)	Kurtosis	Skewness	Min	Max	Num of Obser
HangSeng($)	0.5138	0.1822	93.05	-4.13	-0.4054	0.1725	3390
Nikkei($)	0.1818	0.1606	12.03	-0.04	-0.1791	0.1367	3390
US-S&P	0.5218	0.1015	85.83	-3.81	-0.2283	0.0871	3390

year 2007-2012. In this section, examines how the risk measures perform in these two periods.

Table 1 reports the main descriptive statistics for each of these indices of period 1986 to 1998 in U.S. dollar terms. All three indices have negative skewness, which indicates the tail on the left of the probability density function is longer than the right side. Moreover, asset with negative skewness has a higher downside risk if the excess kurtosis is higher. From Table 1, it can be observed that the Hang-Seng index obtains the highest ex kurtosis and lowest skewness and has the highest downside risk among the three.

Table 2 presents the summary statistics of period 1986 to 2006. Three stock indices continuously obtain negative skewnesses. The biggest difference with table 1 is that S&P 500 possesses the highest excess kurtosis instead, which implies it obtains the highest downside risk in this period.

Table 2

Summary statistics for period 1986-2006

	Mean (x1000)	*Std Dev(x10)*	*Kurtosis*	*Skewness*	*Min*	*Max*	*Num of Obser*
HangSeng($)	0.4427	0.1651	86.42	-3.48	-0.4054	0.1725	5476
Nikkei($)	0.1437	0.1569	9.58	-0.05	-0.1791	0.1367	5476
US-S&P	0.3379	0.0928	445.68	-12.84	-0.3488	0.0871	5476

In table 3, the first three columns are reported in percentage to identify how much of total wealth gets invested in each asset. The forth column is the quantile-α estimated with fixing the order statistics at a small fraction of the sample (approximately 0.29%), say setting $\mathrm{m} = 10$, and choosing $\mathrm{p} = 0.00025$ to be the probability being expected. Then, to obtain the risk premium over return opportunity loss which is the objective function I want to max, I apply two values for the risk free rate. Risk free rate is the average Treasury bill rate. The two values for the risk free rate I pick are the historical risk free rate and a simple zero (i.e. $r = 0$.) The historical annual risk free rate for year 1998 is 4.73%, and the daily risk-free rate is 0.000178. The sixth and seventh columns respectively give the results with two values of r.

Different risk free rates do have impacts on the maximization problem. This verdict can be verified by looking at the top five portfolios (have double or trip * sign on their right sides) selected in Table 3 in columns six and seven: different risk free rates, in fact, differ the results of the portfolio selection problem in the sense that the top five

Table 3

Safety first portfolio selection using VaR of year 1986-1998

Hang Seng	*Nikkei*	*S&P500*	q_α	*Mean(*1000)*	*R-r/r-q*	*R/-q*
10	10	80	-0.06236	0.487021	0.004941**	0.00780926**
10	20	70	-0.0694	0.453018	0.003953	0.00652759
10	30	60	-0.06133	0.419016	0.003918	0.00683184
10	40	50	-0.06464	0.385014	0.003194	0.00595616
10	50	40	-0.08163	0.351012	0.002115	0.0043003
10	60	30	-0.08961	0.317010	0.001548	0.00353777
10	70	20	-0.09193	0.283008	0.00114	0.00307861
10	80	10	-0.09512	0.249006	0.000745	0.00261794
20	10	70	-0.0703	0.486217	0.004373**	0.006916
20	20	60	-0.06443	0.452215	0.004245	0.00701919
20	30	50	-0.05698	0.418213	0.004203	0.00733979**
20	40	40	-0.06054	0.384211	0.003396	0.00634659
20	50	30	-0.07312	0.350209	0.00235	0.00478978
20	60	20	-0.08614	0.316207	0.001601	0.00367103
20	70	10	-0.09352	0.282205	0.001112	0.00301771
30	10	60	-0.06332	0.485414	0.004842**	0.00766642**
30	20	50	-0.06258	0.451412	0.004357	0.00721358
30	30	40	-0.05722	0.417410	0.004171	0.00729515
30	40	30	-0.07287	0.383408	0.002812	0.00526131
30	50	20	-0.07413	0.349406	0.002307	0.00471311
30	60	10	-0.08354	0.315404	0.001641	0.00377528
40	10	50	-0.06526	0.484610	0.004686**	0.00742596**
40	20	40	-0.05498	0.450608	0.004943***	0.00819655***
40	30	30	-0.05783	0.416606	0.004113	0.00720355

Table 3 Continued

Hang Seng	*Nikkei*	*S&P500*	q_α	*Mean(*1000)*	*R-r/r-q*	*R/-q*
40	40	20	-0.07297	0.382604	0.002797	0.00524297
40	50	10	-0.07762	0.348602	0.002193	0.00449127
50	10	40	-0.08374	0.483807	0.003644	0.00577783
50	20	30	-0.07368	0.449805	0.00368	0.00610476
50	30	20	-0.07879	0.415803	0.003011	0.00527734
50	40	10	-0.08325	0.381801	0.002443	0.0045863
60	10	30	-0.10518	0.483004	0.002895	0.00459209
60	20	20	-0.08926	0.449002	0.00303	0.00503024
60	30	10	-0.09525	0.415000	0.002484	0.00435706
70	10	20	-0.10298	0.482200	0.002949	0.00468235
70	20	10	-0.10054	0.448198	0.002683	0.00445782
80	10	10	-0.11295	0.481397	0.002682	0.00426214

portfolios given by the two risk free rates are not all the same. However, it can be clearly observed that the best portfolios selected by the two risk free rates are the same in the tech boom. This phenomenon implies that the risk free rate does not address significant different in the safety first portfolio selection problem. Since then, the investor may concern to invest in the best portfolio given by $r = 0.000178$ which is an investment of 40% in Hang-Seng Index, 20% in Nikkei and 40% in S&P 500, and see how it performed during 1999 to 2003.

To compute the portfolio's expected return during the tech boom, one first need to obtain the rate of return on the three assets that construct the portfolio. In this case, the rate of return on an asset during day Jan 1st 1999 and day Dec 31st 2003 is computed as the log differences of price indexes of the beginning and ending dates. For example, the price

Table 4

Mean-shortfall with the natural estimation of ES of period 1986-1998

Hang Seng	*Nikkei*	*S&P 500*	$s_{0.05}$	$s_{0.0029}$	*Hang Seng*	*Nikkei*	*S&P 500*	$s_{0.05}$	$s_{0.0029}$
10	**10**	**80**	0.021157	0.07017	**30**	**40**	**30**	0.024128	0.074634
10	**20**	**70**	0.020251***	0.064005	**30**	**50**	**20**	0.026394	0.071894
10	**30**	**60**	0.02028**	0.058996**	**30**	**60**	**10**	0.029151	0.07152
10	**40**	**50**	0.021298	0.056767***	**40**	**10**	**50**	0.023527	0.0729
10	**50**	**40**	0.023134	0.057647**	**40**	**20**	**40**	0.023797	0.075404
10	**60**	**30**	0.025561	0.059571**	**40**	**30**	**30**	0.024939	0.085403
10	**70**	**20**	0.028518	0.06286**	**40**	**40**	**20**	0.026703	0.083205
10	**80**	**10**	0.031684	0.066495	**40**	**50**	**10**	0.029038	0.081008
20	**10**	**70**	0.021022**	0.07324	**50**	**10**	**40**	0.0261	0.081593
20	**20**	**60**	0.020438**	0.068434	**50**	**20**	**30**	0.026756	0.083487
20	**30**	**50**	0.020884**	0.064673	**50**	**30**	**20**	0.028067	0.09497
20	**40**	**40**	0.022306	0.063788	**50**	**40**	**10**	0.02988	0.092772
20	**50**	**30**	0.024407	0.064741	**60**	**10**	**30**	0.029303	0.091204
20	**60**	**20**	0.02706	0.066685	**60**	**20**	**20**	0.030129	0.092721
20	**70**	**10**	0.030138	0.069421	**60**	**30**	**10**	0.031543	0.104537
30	**10**	**60**	0.021877	0.078348	**70**	**10**	**20**	0.032845	0.102655
30	**20**	**50**	0.021679	0.07017	**70**	**20**	**10**	0.033747	0.102243
30	**30**	**40**	0.0225	0.064005	**80**	**10**	**10**	0.036562	0.114615

index of S&P 500 on Jan 1st 1999 is 1229.23 dollars, and on Dec 31st 2003 is 1111.92 dollars. Thus, the rate of return of S&P 500 is $ln(\frac{1229.23}{1111.92})$, i.e. -0.1003. For Nikkei and Hang-Seng, the exchange rates of Japan Yen and Hong Kong $ are also applied in the calculation of the rates of returns. During the tech boom, Hang-Seng had a positive return of 0.222074, and Nikkei made a loss of -0.20304. By summing up rate of returns (in US $) with the fixed proportions, the investor will get his or her portfolio return during the announced period. In this case, the investor would gain a selected portfolio return of 0.008103 during the tech boom if he believed the safety first portfolio selection problem using the VaR as the risk measure.

Table 4 represents the results of the mean-shortfall portfolio selection problem with the natural computation of expected shortfall depending on the data of period 1986 to 1999. Columns one to three and five to seven identify the fraction of each asset; the rest columns give the estimation of $s_\alpha(\gamma)$ which is the objective variable the mean-shortfall optimization problem seeks to minimize. I do not put in the column of the portfolio's mean. Since with the same proportions of assets and the same time period, the portfolio's mean would be the same as the ones presented in Table 3.

The estimation of $s_\alpha(\gamma)$ depends on the value of α chosen. Here, I choose two values of α: $\alpha = 0.05$ and $\alpha = 0.0029$. By marking the smallest $s_\alpha(\gamma)$ and top five optimal portfolios as well, the impact of different α can be seen. The insight of α is that the lower the α is, the less willing will be for the investor to obtain the indices with higher downside risks. Referred back to Table 1, Hang-Seng has the highest downside risk in the period before the tech boom. Thus the two values of α both choose to obtain very little proportion on the Hang-Seng. Meanwhile, the smaller α tends to be stricter on the

Table 5

Mean-shortfall with kernel estimation of ES of period 1986-1998

Hang Seng	*Nikkei*	*S&P 500*	$s_{0.05}$	$s_{0.00029}$	$s_{0.00025}$
10	**10**	**80**	0.000493	-0.006352	-0.009477
10	**20**	**70**	0.000447	0.005866	0.009231
10	**30**	**60**	0.000330	0.006455	0.004041
10	**40**	**50**	0.000513	-0.000288	-0.004968
10	**50**	**40**	0.000373	0.011740	0.011049
10	**60**	**30**	0.000321	-0.008183	-0.010703
10	**70**	**20**	0.000157**	0.007711	0.012123
10	**80**	**10**	-0.000116***	0.006706	-0.011728**
20	**10**	**70**	0.000465	-0.002260	0.006151
20	**20**	**60**	0.000458	-0.006646	-0.006753
20	**30**	**50**	0.000391	0.003901	0.004458
20	**40**	**40**	0.000369	0.009259	0.011965
20	**50**	**30**	0.000360	-0.006018	0.012435
20	**60**	**20**	0.000277**	0.005017	0.005770
20	**70**	**10**	0.000254**	0.008909	0.000280
30	**10**	**60**	0.000497	0.000486	-0.007616
30	**20**	**50**	0.000586	-0.006183	-0.007245
30	**30**	**40**	0.000451	-0.006826	-0.007985
30	**40**	**30**	0.000369	0.007859	0.009055
30	**50**	**20**	0.000372	-0.007497	-0.006916

Table 5 Continued

Hang Seng	*Nikkei*	*S&P 500*	$s_{0.05}$	$s_{0.00029}$	$s_{0.00025}$
30	**60**	**10**	0.000294**	-0.011470***	0.016133
40	**10**	**50**	0.000509	-0.005617	-0.011932**
40	**20**	**40**	0.000464	-0.008484	-0.009914
40	**30**	**30**	0.000675	-0.010036**	-0.011709
40	**40**	**20**	0.000435	0.000382	0.012309
40	**50**	**10**	0.000342	0.011367	0.014138
50	**10**	**40**	0.000515	-0.009281	-0.009331
50	**20**	**30**	0.000924	-0.005977	-0.007005
50	**30**	**20**	0.000445	-0.009268	-0.015211***
50	**40**	**10**	0.000391	-0.010823**	-0.012616**
60	**10**	**30**	0.000407	0.006148	0.007054
60	**20**	**20**	0.000403	0.014874	-0.007266
60	**30**	**10**	0.000421	-0.003480	-0.010507
70	**10**	**20**	0.000558	-0.005953	-0.011291
70	**20**	**10**	0.000673	-0.009720**	-0.011167
80	**10**	**10**	0.000500	-0.011335**	-0.013226**

fractions of Hang-Seng in its top five optimal portfolios than the larger α that the top five optimal portfolios all maintain the lowest fraction which is allowed.

Finding the minimum $s_\alpha(\gamma)$ to get the optimal portfolio, the investor ends up with investing 10% in Hang-Seng Index, 20% in Nikkei and 70% in S&P 500 when $\alpha = 0.05$. The portfolio return in the following five years is -0.0886. The portfolio selected by $\alpha = 0.0029$ gives a lower return than $\alpha = 0.05$ does. Unfortunately, comparing to the portfolio return chosen by safety first with VaR, the mean-shortfall with the natural expected shortfall does worse since it is giving a return of negative 0.08861 during 1999 to 2003.

Table 5 is the mean-shortfall optimization problem with kernel-weighted estimator during the tech boom period. Three values of α: $\alpha = 0.05$, $\alpha = 0.0029$ and $\alpha = 0.0025$ are presented. With $\alpha = 0.05$, the portfolio allocates 10% of its wealth in Hang-Seng, 80% of its wealth in Nikkei and 10% in S&P 500. With $\alpha = 0.0029$, an invest of 30% in Hang-Seng, 60% in Nikkei and 10% in S&P 500 has been chosen. With $\alpha = 0.0025$, the mean-shortfall method with the kernel-weighted estimator generates a portfolio of 80% in Hang-Seng, 10% in Nikkei and 10% in S&P 500. From the historical data of 1999 to 2003, Hang-Seng Index is the only asset that made a positive return during this period. This implies that the higher proportion assigned to Hang-Seng Index, the higher return would result in the tech boom period. Thus, the portfolio given by $\alpha = 0.0025$ with the kernel-weighted estimator is the best portfolio that would give the highest return during the tech boom. Also, under the mean-shortfall portfolio selection problem with the kernel estimation, a smaller α generates a better portfolio to a certain level during the tech boom.

Thus, looking at Table 6, it shows that the mean-shortfall portfolio selection problem with the smaller α using the kernel-weighted estimator performs better than the safety first portfolio selection, while safety first portfolio selection performs better than the mean-shortfall portfolio selection problem with the larger α using the natural estimator during the tech boom.

Now I want to take a look at the top five optimal portfolios given by these three methods. The safety first criterion and the mean-shortfall with the kernel estimation has a more diversified selection on their top five portfolios, while the mean-shortfall with the

Table 6

1999-2003 Return

Risk measures		*Return*
VaR	R-r/r-q	0.008103
	R/-q	0.008103
Natural estimation of ES	s0.05	-0.08861
	s0.0029	-0.10916
Kernel estimation of ES	s0.05	-0.15025
	s0.0029	-0.06523
	s0.0025	0.030066

natural estimation seems to be not diversified enough between its top five selected portfolios.

Table 7 has the same structure as Table 3. Table 7 represents the data analysis result of the period before the financial crisis using the safety first. Still choose $p = 0.00025$ to be the probability, and two values of the risk free rate have been examined. For year 2006 the annual rate free rate is 4.68%. Accordingly, risk free rate used to calculate $\frac{\bar{R}-r}{r-q_{\alpha}(R)}$ is 0.000182. I also construct portfolios with a zero risk free rate, and again mark the top five optimal choices. It again claims that the risk free rate matters in the safety first portfolio selection problem by looking at the top five portfolios chosen by the different risk free rates. Furthermore, by noticing that the two values of risk free rate has assigned the same optimal portfolio, it again proves that the impact of risk free rate on safety first criterion is not significant. Concluding on this, the investor can focus on either the case computed from the actual risk free rate or the case computed from a zero risk free rate.

Table 7

Safety first portfolio selection using VaR of year 1986-2006

Hang Seng	*Nikkei*	*S&P 500*	*q*	*Mean(*1000)*	*R-r/r-q*	*R/-q*
10	**10**	**80**	-0.05134	0.32896	0.002852**	0.006408**
10	**20**	**70**	-0.0586	0.309544	0.00217	0.005282
10	**30**	**60**	-0.05145	0.290127	0.002094	0.005639
10	**40**	**50**	-0.06689	0.270711	0.001323	0.004047
10	**50**	**40**	-0.07038	0.251295	0.000982	0.003571
10	**60**	**30**	-0.07391	0.231878	0.000673	0.003137
10	**70**	**20**	-0.08128	0.212462	0.000374	0.002614
10	**80**	**10**	-0.09274	0.193046	0.000119	0.002082
20	**10**	**70**	-0.05279	0.339438	0.002972***	0.00643***
20	**20**	**60**	-0.05681	0.320022	0.002422	0.005634
20	**30**	**50**	-0.05127	0.300606	0.002305	0.005863**
20	**40**	**40**	-0.06502	0.281189	0.001521	0.004324
20	**50**	**30**	-0.06982	0.261773	0.00114	0.00375
20	**60**	**20**	-0.08695	0.242357	0.000693	0.002787
20	**70**	**10**	-0.08786	0.22294	0.000465	0.002537

Table 7 Continued

Hang Seng	*Nikkei*	*S&P 500*	*q*	*Mean(*1000)*	*R-r/r-q*	*R/-q*
30	**10**	**60**	-0.06212	0.349917	0.002695**	0.005633
30	**20**	**50**	-0.05513	0.3305	0.002685	0.005995**
30	**30**	**40**	-0.06499	0.311084	0.001981	0.004786
30	**40**	**30**	-0.07306	0.291668	0.001497	0.003992
30	**50**	**20**	-0.07431	0.272251	0.001212	0.003664
30	**60**	**10**	-0.0857	0.252835	0.000825	0.00295
40	**10**	**50**	-0.06374	0.360395	0.002791**	0.005654**
40	**20**	**40**	-0.06944	0.340979	0.002283	000491
40	**30**	**30**	-0.07373	0.321562	0.001888	0.004362
40	**40**	**20**	-0.07108	0.302146	0.001686	0.00425
40	**50**	**10**	-0.07959	0.28273	0.001263	0.003552
50	**10**	**40**	-0.06897	0.370873	0.002731**	0.005377
50	**20**	**30**	-0.07841	0.351457	0.002156	0.004482
50	**30**	**20**	-0.07462	0.332041	0.002006	0.00445
60	**10**	**30**	-0.08708	0.381352	0.002284	0.004379
60	**20**	**20**	-0.08378	0.361935	0.002143	0.00432
70	**10**	**20**	-0.10599	0.39183	0.001976	0.003697
70	**20**	**10**	-0.09208	0.372414	0.002064	0.004044
80	**10**	**10**	-0.12083	0.402308	0.001821	0.00333

The optimal portfolio under safety first portfolio selection problem with the actual risk free rate is given by 20% investing in Hang-Seng index, 10% investing in Nikkei and 70% investing in S&P 500. During the financial crisis, both Hang-Seng and S&P 500 have positive rates of returns. Furthermore, Hang-Seng gained a higher return at 0.054673 than that of S&P 500 at 0.023732. Nikkei continued to experience a loss of

0.21231. Thus, by February 2012, investing in such a portfolio will bring the investor a profit with a positive return of 0.006316.

Table 8 represents the portfolio selection problem by mean-shortfall method for period 1986 to 2006. Again, I arbitrarily pick two values of α: 5% and 0.25%. The choice of α impacts the choice of the optimal portfolio. Now I mainly concern about the optimal portfolio with probability level $\alpha = 5\%$. It suggests the investor to allocate the assets as $(10\%, 60\%, 30\%)$ in Hong Kong, Japan and U.S. indices such that it provides a negative return of 0.1148 at the end of the financial crisis period. Recall the summary

Table 8

Mean-shortfall with natural estimation of ES of period 1986-2006

Hang Seng	*Nikkei*	*S&P 500*	$s_{0.00025}$	$s_{0.05}$	*Hang Seng*	*Nikkei*	*S&P 500*	$s_{0.00025}$	$s_{0.05}$
10	**10**	**80**	0.280256	0.029055	**30**	**40**	**30**	0.166975	0.019028
10	**20**	**70**	0.245245	0.024303	**30**	**50**	**20**	0.163097	0.018065**
10	**30**	**60**	0.210235	0.025117	**30**	**60**	**10**	0.159219	0.018172**
10	**40**	**50**	0.175225	0.023175	**40**	**10**	**50**	0.210521	0.021537
10	**50**	**40**	0.140215	0.027337	**40**	**20**	**40**	0.206643	0.02268
10	**60**	**30**	0.105204***	0.018287**	**40**	**30**	**30**	0.202765	0.020699
10	**70**	**20**	0.115552**	0.017734***	**40**	**40**	**20**	0.198886	0.019403
10	**80**	**10**	0.13857	0.0244	**40**	**50**	**10**	0.195008	0.019026
20	**10**	**70**	0.245275	0.025944	**50**	**10**	**40**	0.242432	0.023231
20	**20**	**60**	0.210265	0.027872	**50**	**20**	**30**	0.238554	0.027543
20	**30**	**50**	0.175255	0.025902	**50**	**30**	**20**	0.234675	0.025098

Table 8 Continued

Hang Seng	*Nikkei*	*S&P 500*	$s_{0.00025}$	$s_{0.05}$	*Hang Seng*	*Nikkei*	*S&P 500*	$s_{0.00025}$	$s_{0.05}$
20	**40**	**40**	0.140245	0.030427	**50**	**40**	**10**	0.230797	0.023081
20	**50**	**30**	0.131186**	0.021818	**60**	**10**	**30**	0.274343	0.02738
20	**60**	**20**	0.127308**	0.019667	**60**	**20**	**20**	0.270465	0.026198
20	**70**	**10**	0.123429**	0.018206**	**60**	**30**	**10**	0.266586	0.027958
30	**10**	**60**	0.210295	0.020771	**70**	**10**	**20**	0.306254	0.030612
30	**20**	**50**	0.175285	0.028911	**70**	**20**	**10**	0.302375	0.029402
30	**30**	**40**	0.170854	0.02083	**80**	**10**	**10**	0.338165	0.03273

statistics of the three assets before the financial crisis in Table 2, Nikkei obtain the smallest downside risk among the three indices before the financial crisis, and thus it is preferable in this mean-shortfall portfolio optimization problem with the natural

Table 9

Mean-shortfall with kernel estimation of ES of period 1986-2006

Hang Seng	*Nikkei*	*S&P 500*	$s_{0.05}$	$s_{0.0029}$	$s_{0.0025}$
10	**10**	**80**	0.000566	-0.004715	-0.004783
10	**20**	**70**	0.000313	-0.005722	-0.006611
10	**30**	**60**	0.000290	0.000290	0.000290
10	**40**	**50**	0.000385	-0.001572	-0.001867
10	**50**	**40**	0.000266	-0.008732**	-0.007620
10	**60**	**30**	0.000238**	-0.004805	-0.004668
10	**70**	**20**	0.000075***	0.001549	0.001763
10	**80**	**10**	0.000193**	0.000193	0.000193

Table 9 Continued

Hang Seng	*Nikkei*	*S&P 500*	$s_{0.05}$	$s_{0.0029}$	$s_{0.0025}$
20	**10**	**70**	0.000372	-0.004863	-0.005735
20	**20**	**60**	0.000593	-0.005270	0.009004
20	**30**	**50**	0.000294	0.006996	-0.006676
20	**40**	**40**	0.000290	-0.003810	-0.004342
20	**50**	**30**	0.000268	-0.004088	-0.004783
20	**60**	**20**	0.000235**	0.005893	0.006797
20	**70**	**10**	0.000206**	0.005094	0.005873
30	**10**	**60**	0.000382	-0.005944	0.003615
30	**20**	**50**	0.000613	-0.004320	-0.005173
30	**30**	**40**	0.000332	-0.000521	-0.000654
30	**40**	**30**	0.000296	-0.004927	-0.006019
30	**50**	**20**	0.000283	-0.002733	-0.003213
30	**60**	**10**	0.000246	0.007712	0.007551
40	**10**	**50**	0.000396	-0.005666	-0.007508
40	**20**	**40**	0.000357	-0.002178	-0.002581
40	**30**	**30**	0.000586	-0.006366	-0.005749
40	**40**	**20**	0.000334	-0.005024	-0.001803
40	**50**	**10**	0.000288	-0.006796**	-0.007928**
50	**10**	**40**	0.000386	0.002874	-0.003470
50	**20**	**30**	0.000406	-0.000285	-0.000387
50	**30**	**20**	0.000338	-0.007746**	-0.009038**
50	**40**	**10**	0.000313	0.000388	0.000313
60	**10**	**30**	0.000357	0.004926	0.005654
60	**20**	**20**	0.000363	0.000362	0.000362
60	**30**	**10**	0.000353	-0.010087**	-0.009315**

Table 9 Continued

Hang Seng	*Nikkei*	*S&P 500*	$s_{0.05}$	$s_{0.0029}$	$s_{0.0025}$
70	**10**	**20**	0.000397	-0.010541***	-0.011144***
70	**20**	**10**	0.000604	0.010279	-0.009772**
80	**10**	**10**	0.000398	0.013228	0.013863

estimator. Referred back to Table 2, in the period before the financial crisis S&P 500 has the highest downside risk. Thus, S&P 500 is less preferable than the other two assets. The top five selections under this method again suggest that this method tends to put less investment in the indices with high downside risks. This phenomenon further reinforced that under this method, the optimal portfolio is chosen in a way that deeply influenced by the choice of α.

Table 10

2007-2012 Return

Risk measures		*Return*
VaR	R-r/r-q	-0.0461
	R/-q	-0.0461
Natural estimation of ES	s0.0025	-0.1297
	s0.05	-0.13998
Kernel estimation of ES	s0.05	-0.13998
	s0.0029	0.115088
	s0.0025	0.115088

Table 9 is the layout of the mean-shortfall optimization problem using the kernel-weighted estimator for year 1987 to 2006. Three values of α: 5%, 0.29% and 0.25% have been experienced. The impact of α during the financial crisis is similarly to that during the tech boom: a smaller α results in a better choice of portfolio. This can be seen from Table 10. Table 10 is the return table.

Note that, in the financial crisis, α=0.29% and α=0.25% give the same best portfolio of 70% in Hang-Seng, 10% in Nikkei, and 20% in S&P 500. As shown above, the smaller the α is, the more efficient the mean-shortfall with the kernel estimation is, up to a certain level. Comparing to previous two methods, this method assigns the optimal portfolio with the highest return of 0.021786 during the financial crisis.

The three methods obtain the same pattern of performance in both the tech boom and the finical crisis circumstances: the mean-shortfall with the kernel estimation is better than the safety first, and the safety first is better than the mean-shortfall with the natural estimation.

The main reason causing mean-shortfall methods differ under two estimations is the effectiveness of the estimation on the expected shortfall. The effectiveness of the expected shortfall depends on the stability of the estimation. The kernel-weighted estimation is more stable than the natural estimation. One problem of the mean-shortfall optimization with the natural estimation is that it is the average of VaRs for all levels below α. This implies that mean-shortfall portfolio optimization draws heavily on the worst α cases. Thus, it also depends heavily on the choice of α in the sense that which worse cases have been considered. For this reason, it is not true for the mean-shortfall with the natural estimation that smaller α generates better portfolio. In other words, the impact of α is not stable under the natural estimation. As a result, in this practical sample, it assigns little weights to Hang-Seng Index and S&P 500 which made profits during the financial crisis.

Theoretically speaking, expected shortfall is considered to be a more sensitive risk measure than VaR to the loss distribution. Thus, the mean-shortfall should perform better in the financial crisis circumstance. The mean-shortfall optimization problem with the kernel estimation does perform better than other two methods. Not only that, it generated a relatively higher return to the safety first did during the financial crisis period than the tech boom. It has been shown that the mean-shortfall with the stable estimation (kernel) of expected shortfall is always better than the safety first portfolio selection problem.

6. CONCLUSION

Empirically, I show that the safety first portfolio selection with VaR is more suitable than the mean-shortfall portfolio optimization with the natural estimation, and less effective than the mean-shortfall portfolio optimization with the kernel estimation. Both the safety first portfolio selection problem and the mean-shortfall with stable estimation (kernel estimation) perform well under the tech boom, and the financial crisis situations, whereas the mean-shortfall optimization with the kernel estimation, is in general a better method than the safety first criterion. Moreover, the mean-shortfall with a stable estimation is a more effective method in the financial crisis period. Thus, a risk averse investor who worries about future economic situations for should always choose his optimal portfolio based on the mean-shortfall portfolio selection problem using a stable estimation instead of the safety first criterion.

REFERENCES

Acerbi, C. and Tasche, D. (2001). ‘Expected shortfall: a natural coherent alternative to value at risk’, *Economic Notes*, vol. 31(2), pp. 1-10.

Artzner, P., Delbaen, F., Eber, J.M. and Heath, D. (1999). ‘Coherent measure of risk’, *Mathematical Finance*, vol. 9(3), pp. 203–28.

Arzac, E.R. and Bawa, V.S. (1977). ‘Portfolio choice and equilibrium in capital markets with safety-first investors’, *Journal of Financial Economics,* vol. 4(3), pp. 277-88.

Bertsimas, D., Lauprete, G.J. and Samarov, A. (2004) ‘Shortfall as a risk measure: properties, optimization and applications’, *Journal of Economic Dynamics & Control*, vol. 28(7), pp. 1353–81.

Haan, D.L., Jansen, D.W., Koedijk, K. and de Vries, C.G. (1994). ‘Safety first portfolio selection, extreme value theory and long run asset risks’, in J. Galambos, J. Lechner, and E. Simiu (eds), *Extreme Value Theory and Applications: Proceedings from a Conference on Extreme Value Theory and Applications*, Netherlands, Dordrecht: Kluwer Academic Publishing, pp.471-87.

Fan, J. and Yao, Q. (2003). *Nonlinear Time Series: Nonparametric and Parametric Methods*, New York: Springer.

Galambos, J., Lechner, J. and Simiu,E. (1994). *Extreme Value Theory and Applications: Proceedings from a Conference on Extreme Value Theory and Applications*, Netherlands, Dordrecht: Kluwer Academic Publishing.

Hill, B.M. (1975). ‘A simple general approach to inference about the tail of a distribution’, *The Annuals of Statistics.* vol. 3(5), pp. 1163-73.

Jansen, D.W. (2001). ‘Limited downside risk in portfolio selection among U.S. and pacific basin equities’, *International Economics Journal*, vol. 15(4), pp. 1-39.

Jansen, D.W. and de Vries, C.G. (1991). ‘On the frequency of large stock returns: putting booms and busts into perspective’, *Review of Economics and Statistics,* vol. 73(1), pp. 18-24.

Jansen, D.W., Koedijk, K., and de Vries, C.G. (2000). ‘Portfolio selection with limited downside risk’, *Journal of Empirical Finance,* vol. 7(3-4), pp. 247-70.

Jorion, P. (2006). *Value at Risk: The New Benchmark for Managing Financial Risk* (3rd ed.), New York: McGraw-Hill.

Koedijk, K.G., Schafgans, M.M.A. and de Vries, C.G. (1990). ‘The tail index of exchange rate returns’, *Journal of International Economics,* Vol. 29(1-2), pp. 93-108.

Koponen, T.M. (2003). ‘Commodities in action: measuring embeddedness and imposing values’, *The Sociological Review,* vol. 50(4), pp. 543–69.

Levy, H. (1992). ‘Stochastic dominance and expected utility: survey and analysis’, *Management Science*, vol. 38(4), pp. 555–93.

Levy, H. and Kroll, Y. (1978). ‘Ordering uncertain options with borrowing and lending’ *The Journal of Finance*, vol. 31(2), pp. 553–74.

Longin, F.M. (1996). ‘The asymptotic distribution of extreme stock market returns’, *Journal of Business,* vol. 69(3), pp. 383-408.

Longin, F.M. (2000). ‘From value at risk to stress testing: the extreme value approach’, *Journal of Banking & Finance*, vol. 24(7), pp. 1097-1130.

Longin, F.M. and Solnik B. (2001). ‘Extreme correlation of international equity markets’, *Journal of Finance*, vol. 56(2), pp. 649-76.

Loretan, M. and Phillips P.C.B. (1994). ‘Testing the covariance stationarity of heavy tailed time series’, *Journal of Empirical Finance*, vol. 1(2), pp. 211-48.

Markowitz, H.M. (1952). ‘Portfolio selection’, *The Journal of Finance*, vol. 7(1), pp. 77–91.

Mason, D.M. (1982). ‘Laws of large numbers for sums of extreme values’, *Annals of Probability*, vol. 10(3), pp. 754-64.

Reiss, R.D. and Thomas, M. (1997). *Statistical Analysis of Extreme Values*, Germany, Basel: Birkhäuser.

Roy, A.D. (1952). ‘Safety first and the holding of assets’, *Econometrica*, vol. 20(3), pp. 431-49.

Shleifer, A. (2000). *Inefficient Markets: An Introduction to Behavioral Finance*. Clarendon Lectures, United Kindom, Oxford: Oxford University Press.

Tasche, D. (1999). *Risk contributions and performance measurement*. Report of the Lehrstuhl für mathematische Statistik, TU München

Uryasev, S. and Rockafellar, R.T. (2000). ‘Optimization of conditional value-at-risk’, *Journal of risk*, vol. 2, pp. 21-42.